ENGLISH COLLEGE
ANGLICKÉ GYMNÁZIUM IN PRAGUE

YEAR 3

Mathematics Entrance Examination Practice set (B)
Total marks - 54

FORMULAE SHEET (*only some of these will be needed)

Equation of a line:

Gradient-intercept form: $y = mx + c$ where m is the gradient and c is the y-intercept

General form: $ax + by = d$ where a,b and d are integers

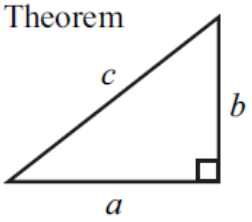
For 2 points (x_1, y_1) and (x_2, y_2) :

The midpoint $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

The distance $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

The gradient $m = \frac{y_2-y_1}{x_2-x_1}$

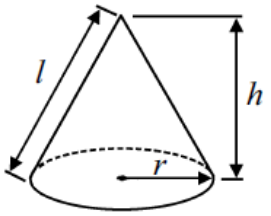
Pythagoras' Theorem



$a^2 + b^2 = c^2$

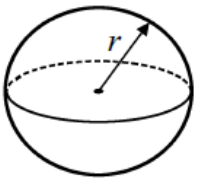
Volume of cone = $\frac{1}{3}\pi r^2 h$

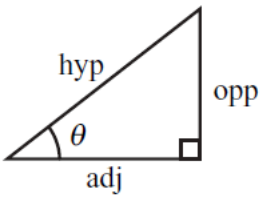
Curved surface area of cone = $\pi r l$



Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



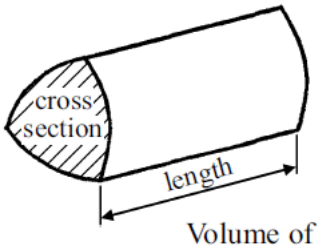


adj = hyp × cos θ
opp = hyp × sin θ
opp = adj × tan θ

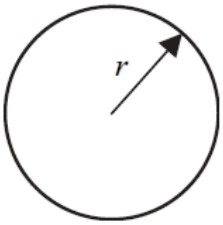
or $\sin\theta = \frac{\text{opp}}{\text{hyp}}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}}$

$\tan\theta = \frac{\text{opp}}{\text{adj}}$



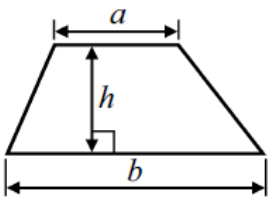
Volume of prism = area of cross section × length

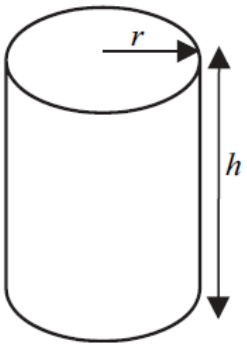


Circumference of circle = $2\pi r$

Area of circle = πr^2

Area of a trapezium = $\frac{1}{2}(a+b)h$





Volume of cylinder = $\pi r^2 h$

Curved surface area of cylinder = $2\pi rh$

The Quadratic Equation
The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Questions

1. 850 people attended a festival. 16% of the people were children
850 lidí se zúčastnilo festivalu. 16% z nich bylo dětí.

a) How many children were at the festival?

Kolik dětí se zúčastnilo festivalu?

Answer (a) ……...…… **[1]**

b) What was the ratio of adults to children at the festival? Simplify your answer as much as possible.

Jaký byl na festivalu poměr mezi dospělými a dětmi? Svou odpověď napiš v základním tvaru.

Answer (b) ……...…… **[2]**

c) The organisers are expecting ticket sales to increase by 12% next year. How many tickets do they expect to sell next year?

Organizátoři očekávají, že prodej vstupenek příští rok vzroste o 12%. Kolik vstupenek plánují příští rok prodat?

Answer (b) ……...…….…… **[2]**

2. A car dealer offers a discount of 15% off the normal price of a car for cash. Emma pays £6 120 cash for a car. What was the normal price of the car if there was no discount?

   Prodejce aut nabízí 15% slevu z běžné ceny, pokud zákazník platí hotově. Emma zaplatí za auto 6 120 liber v hotovosti. Jaká byla původní cena auta bez slevy?

Answer (a) ……...…….…… **[2]**

3. Marek is organising a charity hot dog sale. The bread rolls he buys have 18 rolls in each packet. He buys hot dogs in bags of 15.

   Marek pořádá charitativní prodej hotdogů. Jedno balení obsahuje 18 housek. Párky koupí v baleních po 15 kusech.

   What is the **smallest** number of each packet that he can buy so that he has exactly the same number of bread rolls as hot dogs?

   *Jaký je **nejmenší** možný počet balení housek a párků, které může Marek koupit, pokud chce mít stejné množství housek a párků?*

..... packets of bread rolls

balení housek

..... packets of hot dogs

balení párků

[3]

4. For each of the following sequences write down the next two terms.
V každé číselné řadě uveď dva následující členy.

a) 7, 11, 15, 19, ..., ……..

b) 11, 6, 1, -4, ..., ……..

c) 9, 16, 25, 36, ..., ……..

b) 1 000, 100, 10, 1, ……., ……

[4]

.

5. Solve the equations, showing each step of your workings.
Vyřeš rovnice a soustavu rovnic. Uveď celý postup řešení.

(a) $3(x + 7) = 4(13 - x) + 4$

Answer (a) $x =$ ……

[2]

(b) $x^2 + 4x + 3 = 0$

Answer (b) $x =$ ……. and $x =$ ……

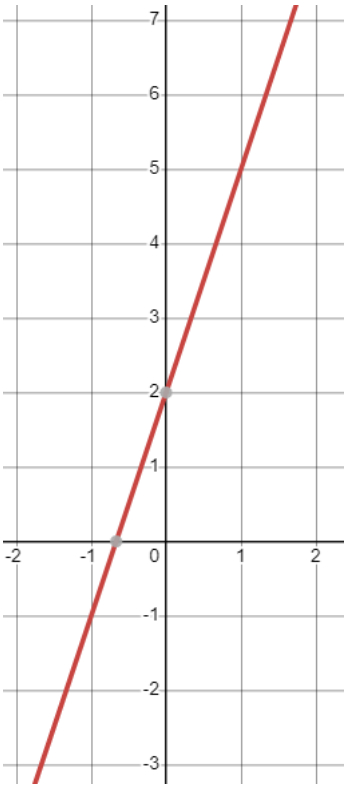
[2]

(c) $x + 4y = 5$ and $4x - 2y = 11$

Answer (c) x = ……..…….. and y = ……..…

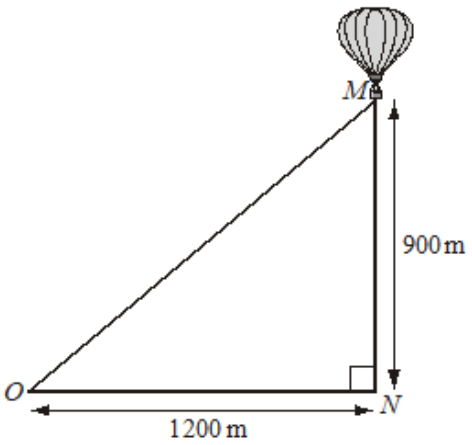
[4]

6. What is the equation of the line shown in the graph?
Jaký je předpis lineární funkce v grafu?



Answer: $y =$ ……**[2]**

7. A hot air balloon, M, is 900 metres vertically above a point N on the ground. A boy stands at a point O, 1200 metres horizontally from N.
Horkovzdušný balón M je 900 metrů vertikálně nad bodem N, který je na zemi. Chlapec stojí v bodě O, 1200 metrů horizontálně od bodu N.



NOT TO SCALE

(a) Calculate the distance, OM, of the boy from the balloon.
Vypočítej vzdálenost chlapce a balónu (OM).

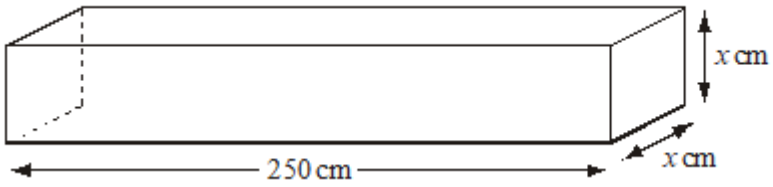
Answer (a) OM = …… m **[2]**

(b) Calculate angle MON.
Vypočítej úhel MON.

Answer (b) Angle MON = ……..

[2]

8. A solid metal bar is in the shape of a cuboid of length 250 cm. The cross-section is a square with side length x centimetres. The volume of the cuboid is 4840 cm^3.
Pevná kovová cihla má tvar kvádru s délkou 250 cm. Průřez kvádru má tvar čtverce se stranou dlouhou x centimetrů. Objem kvádru je 4840 cm^3.



NOT TO SCALE

a) Show that $x = 4.4$ cm. **You must show your workings.**
Dokaž, že x = 4,4 cm. ***Uveď postup řešení.***

b) Find the surface area of the cuboid.
Vypočítej povrch kvádru.

Answer (b) .....**[5]**

9. 18 men build a wall that is 140 metres long in 42 days. How long would it take 30 men to build a wall that is 100 metres long?

18 mužů postaví 140 m dlouhou zeď za 42 dnů. Jak dlouho by trvalo 30 mužům, než by postavili 100 m dlouhou zeď?

Answer .....**[3]**

10.

A rectangular garden has a length that is 3 metres longer than its width, x metres.

A concrete path of uniform width 1 metre is built around all four sides of the garden.

The total area of the garden and the path combined is $70\ \text{m}^2$.

(a) Show that $x^2 + 7x - 55 = 0$.

(b) Solve the equation $x^2 + 7x - 55 = 0$ using the quadratic formula to find the width of the garden, x. Give your answer correct to 2 decimal places.

[6 marks]

11.

Point A has coordinates $(-2, 7)$ and point B has coordinates $(6, 1)$.

(a) Find the coordinates of the midpoint of the line segment AB.

(b) Find the gradient of the line segment AB.

(c) Find the equation of the perpendicular bisector of AB. Give your answer in the form $y = mx + c$.

[7 marks]

12.

Two mathematically similar cones, A and B, have total surface areas of $45\pi \text{ cm}^2$ and $80\pi \text{ cm}^2$ respectively.

(a) Find the ratio of the height of cone A to the height of cone B in its simplest form.

(b) Given that the volume of cone A is 135 cm^3, calculate the volume of cone B.

[5 marks]

THE END / *KONEC*

ARITHMETIC, PERCENTAGES, SEQUENCES & BASIC ALGEBRA (Q1 – Q6)

Q1. Festival Attendance (Percentages & Ratio) [5 Marks]

850 lidí na festivalu, 16% dětí. Poměr dospělí:děti. Nárůst o 12%.

(a) Children (1m): 0.16 × 850 = **136 children**
(b) Adults to Children ratio (2m):
• Adults = 850 − 136 = 714
• Ratio = 714 : 136 = (714÷34) : (136÷34) = **21 : 4**
(c) 12% ticket sales increase (2m):
• 850 × 1.12 = **952 tickets**

(a) 136 | (b) 21 : 4 | (c) 952 tickets

Q2. Reverse Percentage (Car Discount) [2 Marks]

Sleva 15% při platbě v hotovosti. Cena po slevě je £6 120. Původní cena?

Discount of 15% ⇒ £6 120 represents 85% (0.85) of normal price:
• Normal price = £6 120 ÷ 0.85 = **£7 200**

Normal Price = £7 200

Q3. Lowest Common Multiple (Hot Dog Packets) [3 Marks]

Housky v balení po 18, párky po 15. Nejmenší počet balení pro stejný počet?

Find LCM(18, 15):
• 18 = 2 × 3², 15 = 3 × 5
• LCM = 2 × 3² × 5 = 90 items
• Bread rolls packets = 90 ÷ 18 = **5 packets**
• Hot dogs packets = 90 ÷ 15 = **6 packets**

5 packets of rolls, 6 packets of hot dogs

Q4. Number Sequences [4 Marks]

Uveď dva následující členy v každé číselné řadě.

(a) 7, 11, 15, 19... (+4): 23, 27
(b) 11, 6, 1, −4... (−5): −9, −14
(c) 9, 16, 25, 36... (Squares n²): 49, 64
(d) 1000, 100, 10, 1... (÷10): 0.1, 0.01

(a) 23, 27 | (b) −9, −14 | (c) 49, 64 | (d) 0.1, 0.01

Q5. Equations & Simultaneous Systems [8 marks]

(a) Linear Solve (2m):
3(x + 7) = 4(13 − x) + 4
3x + 21 = 52 − 4x + 4
3x + 4x = 56 − 21
7x = 35 ⇒ **x = 5**

(b) Quadratic Solve (2m):
x² + 4x + 3 = 0
(x + 3)(x + 1) = 0
x = −3, x = −1

(c) Simultaneous (3m):
1) x + 4y = 5 ⇒ x = 5 − 4y
2) 4(5 − 4y) − 2y = 11
20 − 18y = 11 ⇒ 18y = 9 ⇒ **y = 0.5**
x = 5 − 4(0.5) = **3**

(a) x = 5 | (b) x = −3, −1 | (c) x = 3, y = 0.5

Q6. Equation of Straight Line from Graph [2 Marks]

Určete předpis lineární funkce z grafu.

• Y-intercept (0, c) = (0, 2) ⇒ c = 2
• Second point read from grid: (−1, 0) or (1, 5)
• Gradient m = (5 − 2) ÷ (1 − 0) = 3 ÷ 1 = 3
• Equation: **y = 3x + 2**

y = 3x + 2

TRIGONOMETRY, 3D GEOMETRY, PROPORTION & ADVANCED TOPICS (Q7 – Q12)

Q7. Pythagoras & Trig [4 Marks]

Balón 900m vysoko, 1200m horizontálně.

(a) Distance OM (2m):
OM = $\sqrt{(1200^2 + 900^2)}$
OM = $\sqrt{(1440000 + 810000)}$
OM = $\sqrt{2250000}$ = **1500 m**
(b) Angle MON (2m):
tan(MON) = 900 ÷ 1200 = 0.75
Angle MON = $\tan^{-1}(0.75) \approx$ **36.9°**

(a) 1500 m | (b) 36.9°

Q8. Cuboid Volume & Area [5 Marks]

Délka 250 cm, čtvercový průřez x, objem 4840 cm³.

(a) Show x = 4.4 cm (2m):
Volume = x × x × 250 = 4840
$250x^2 = 4840 \Rightarrow x^2 = 19.36$
x = $\sqrt{19.36}$ = **4.4 cm**
(b) Surface Area (3m):
2 ends: 2 × (4.4 × 4.4) = 38.72 cm²
4 sides: 4 × (4.4 × 250) = 4400 cm²
Total Area = **4438.72 cm²**

(a) Shown (4.4 cm) | (b) 4438.72 cm²

Q9. Compound Proportion [3 Marks]

18 mužů postaví 140m za 42 dnů. 30 mužů 100m?

- Rate per man per day = 140 ÷ (18 × 42) = $^{140}/_{756} = {}^{5}/_{27}$ metres/man-day
- Total man-days needed for 100m = 100 ÷ ($^{5}/_{27}$) = 540 man-days
- Days for 30 men = 540 ÷ 30 = **18 days**

Time required = 18 days

Q10. Garden Path Quadratic Equation (Quadratic Formula) [6 Marks]

Zahrada x × (x+3). Cesta široká 1m kolem dokola. Celková plocha 70 m².

(a) Show $x^2 + 7x - 55 = 0$ (3m):
- Outer width = x + 1 + 1 = x + 2
- Outer length = (x + 3) + 1 + 1 = x + 5
- Combined Area = (x + 2)(x + 5) = 70
- $x^2 + 7x + 10 = 70 \Rightarrow$ **$x^2 + 7x - 55 = 0$**

(b) Solve with Quadratic Formula (3m):
a = 1, b = 7, c = −55
$x = [-7 \pm \sqrt{(7^2 - 4(1)(-55))}] \div 2 = [-7 \pm \sqrt{(49 + 220)}] \div 2$
$x = [-7 \pm \sqrt{269}] \div 2 \approx [-7 \pm 16.4012] \div 2$
x = 9.4012 ÷ 2 = **4.70 m** (discard negative root −11.70)

(a) Shown: $x^2 + 7x - 55 = 0$ | (b) Garden Width x = 4.70 m (2 d.p.)

Q11. Perpendicular Bisector & Coordinate Geometry [7 Marks]

Body A(−2, 7) a B(6, 1). Střed, směrnice, osa úsečky AB.

(a) Midpoint M (2m):
M = $(^{(-2+6)}/_{2}, {}^{(7+1)}/_{2})$
M = $(^{4}/_{2}, {}^{8}/_{2})$ = **(2, 4)**

(b) Gradient AB (2m):
m = $^{(1-7)}/_{(6-(-2))}$
m = $^{-6}/_{8}$ = **$-{}^{3}/_{4}$**

(c) Perpendicular Bisector (3m):
- Perpendicular gradient $m_\perp$ = −1 ÷ ($-{}^{3}/_{4}$) = $^{4}/_{3}$
- Sub (2, 4) into y = mx + c:
  4 = ($^{4}/_{3}$)(2) + c ⇒ c = 4 − $^{8}/_{3}$ = $^{4}/_{3}$
- Equation: **$y = {}^{4}/_{3}x + {}^{4}/_{3}$** (or 4x − 3y + 4 = 0)

(a) Midpoint = (2, 4) | (b) Gradient = −¾ | (c) $y = {}^{4}/_{3}x + {}^{4}/_{3}$

Q12. Similar Cones & Volume Scale Factor [5 Marks]

Podobné kužele A a B, povrch 45 cm² a 80 cm². Objem A = 135 cm³.

(a) Height ratio A : B (2m):
- Area scale factor (SF^2) = 80 ÷ 45 = $^{16}/_{9}$
- Linear scale factor (SF) = $\sqrt{(^{16}/_{9})} = {}^{4}/_{3}$
- Height Ratio A : B = **3 : 4**

(b) Volume of Cone B (3m):
- Volume scale factor (SF^3) = $(^{4}/_{3})^3 = {}^{64}/_{27}$
- $Volume_B = Volume_A \times SF^3$
- $Volume_B = 135 \times {}^{64}/_{27} = 5 \times 64 =$ **320 cm³**

(a) Ratio A : B = 3 : 4 | (b) Volume of Cone B = 320 cm³